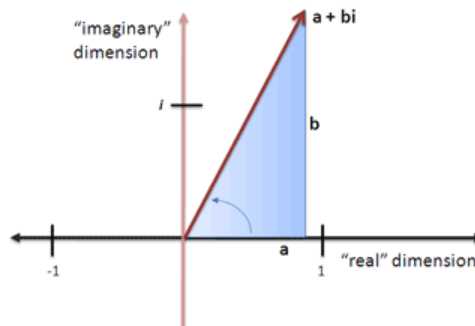


Complex Numbers

Fun Fact	Negative Numbers (-x)	Complex Numbers (a + bi)
Invented to answer	"What is 3 - 4?"	"What is sqrt(-1)?"
Strange because...	<i>How can you have less than nothing?</i>	<i>How can you take the square root of less than nothing?</i>
Intuitive meaning	"Opposite"	"Rotation"
Considered absurd until	1700s	Today 😊
Multiplication cycle [& general pattern]	1, -1, 1, -1... X, -X, X, -X..	1, i, -1, -i... X, Y, -X, -Y..
Use in coordinates	Go backwards from origin	Rotate around origin
Measure size with	Absolute value $\sqrt{(-x)^2}$	Pythagorean Theorem: $\sqrt{a^2 + b^2}$

$a + bi$



Usage of Complex Numbers

1. When designing the control system for the space shuttle, the model was a system of simultaneous equations. When we tested for stability of specific flight conditions (altitude, mach value, angle of attack, etc.) and "broke the loop", we looked at the roots of the equations: if the roots were strictly complex, then the condition was stable; if the roots, however, were real, then the condition was unstable and the design had to be altered. [this work was done at Honeywell Avionics, who were designed the control system]

2. In finding roots of an nth degree polynomial, such as $z^5 = 32$, most algebra students know that there is one real root ($x=2$) and 4 complex roots -- but how do you find the complex roots? If you consider the Cartesian coordinate system as graphing the complex numbers $x + iy$, then the solution of this equation are points on the circle with a radius of 2 (since the norm of z is 2). These roots are equally spaced around the circle (i.e., 72 degrees apart). While this isn't as "real world" application as the previous example, it does a nice job in connecting some mathematical knowledge students may have from algebra, trig, etc.