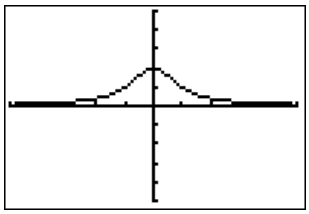
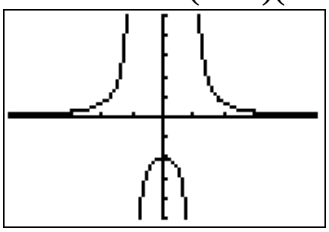
8-3 Notes

Rational Functions and Their Graphs

A. Continuous/Discontinuous Functions

*Focus on denominators because you can NOT divide by ZERO.

Continuous	Discontinuous
No gaps in the graph; functions can be graphed without picking up your pen.	There are gaps in the graph; you must pick up your pen to graph the function.
$f(x)=\frac{2}{x^2+1}$  $x^2+1\neq 0$	$f(x)=\frac{2}{x^2-1}=\frac{2}{(x-1)(x+1)}$  Discontinuous at x = -1 and 1

*If the denominator is not factorable, use the quadratic formula.

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

(If $x = a + bi$, then the function is continuous.)

Examples – Decide if the given function is continuous or discontinuous. If discontinuous, where?

1. $f(x)=\frac{x}{x^2-x-2}$ Discontinuous at x = -1 and 2

2. $y=\frac{x+1}{x^2-3x}$ Discontinuous at x = 0 and 3

3. $y=\frac{2}{x^2+4}$ Continuous (Check with quadratic formula.)

4. $y=\frac{2}{x^2-4}$ Discontinuous at x = -2 and 2

5. $f(x)=\frac{4}{x^4+16}$ Continuous

6. $y=\frac{3}{x^2-5x+6}$ Discontinuous at x = 2 and 3

B. Types of Discontinuities

A rational function may have one or more types of discontinuities: **holes** (removable points of discontinuity), **vertical asymptotes** (non-removable points of discontinuity), or a **horizontal asymptote**.

If	Then	Example
a is a zero that can be *removed*	**hole** at $x = a$	$f(x) = \dfrac{(x-5)(x+6)}{(x-5)}$ hole at $x = 5$
a is a zero that can NOT be *removed*	**vertical asymptote** at $x = a$	$f(x) = \dfrac{x^2}{x-3}$ vertical asymptote at $x = 3$

Let m = degree of numerator.
Let n = degree of denominator.

• $m < n$	horizontal asymptote at $y = 0$	$f(x) = \dfrac{4x}{7x^2 + 2}$
• $m > n$	no horizontal asymptote exists	$f(x) = \dfrac{4x^3}{7x^2 + 2}$
• $m = n$	horizontal asymptote at $y = \dfrac{a}{b}$, where a and b are coefficients of highest degree terms in numerator and denominator	$f(x) = \dfrac{4x^2}{7x^2 + 2}$ horizontal asymptote at $y = \dfrac{4}{7}$

Problem

What are the points of discontinuity of $y = \dfrac{x^2 + x - 6}{3x^2 - 12}$, if any?

Step 1 Factor the numerator and denominator completely. $y = \dfrac{(x-2)(x+3)}{3(x-2)(x+2)}$

Step 2 Look for values that are zeros of both the numerator and the denominator. The function has a hole at $x = 2$.

Step 3 Look for values that are zeros of the denominator only. The function has a vertical asymptote at $x = -2$.

Step 4 Compare the degrees of the numerator and denominator. They have the same degree. The function has a horizontal asymptote at $y = \dfrac{1}{3}$.

Exercises

Find the vertical asymptotes, holes, and horizontal asymptote for the graph of each rational function.

1. $y = \dfrac{x}{x^2 - 9}$

vertical asymptotes: $x = 3$, $x = -3$;
horizontal asymptote: $y = 0$

2. $y = \dfrac{6x^2 - 6}{x - 1}$

hole: $x = 1$

3. $y = \dfrac{4x + 5}{3x + 2}$

vertical asymptote: $x = -\frac{2}{3}$;
horizontal asymptote: $y = \frac{4}{3}$