

6-2

Notes

Multiplying and Dividing Radical Expressions

You can simplify a radical if the radicand has a factor that is a perfect n th power and n is the index of the radical. For example:

$$\sqrt[n]{xy^n z} = y\sqrt[n]{xz}$$

Problem

What is the simplest form of each product?

a. $\sqrt[3]{12} \cdot \sqrt[3]{10}$

$$\sqrt[3]{12} \cdot \sqrt[3]{10} = \sqrt[3]{12 \cdot 10}$$

$$= \sqrt[3]{2^2 \cdot 3 \cdot 2 \cdot 5}$$

$$= \sqrt[3]{2^3 \cdot 3 \cdot 5}$$

$$= \sqrt[3]{2^3} \cdot \sqrt[3]{3 \cdot 5}$$

$$= 2\sqrt[3]{15}$$

$$\text{Use } \sqrt[n]{a} \cdot \sqrt[n]{b} = \sqrt[n]{ab}.$$

Write as a product of factors.

Find perfect third powers.

$$\text{Use } \sqrt[n]{ab} = \sqrt[n]{a} \cdot \sqrt[n]{b}.$$

$$\text{Use } \sqrt[n]{a^n} = a \text{ to simplify.}$$

b. $\sqrt{7xy^3} \cdot \sqrt{21xy^2}$

$$\sqrt{7xy^3} \cdot \sqrt{21xy^2} = \sqrt{7xy^3 \cdot 21xy^2}$$

$$= \sqrt{7xy^2 y \cdot 3 \cdot 7xy^2}$$

$$= \sqrt{7^2 x^2 (y^2)^2 \cdot 3y}$$

$$= 7xy^2 \sqrt{3y}$$

$$\text{Use } \sqrt[n]{a} \cdot \sqrt[n]{b} = \sqrt[n]{ab}.$$

Write as a product of factors.

Find perfect second powers.

$$\text{Use } \sqrt[n]{a^n} = a \text{ to simplify.}$$

Exercises

Simplify each product.

1. $\sqrt{15x} \cdot \sqrt{35x}$

2. $\sqrt[3]{50y^2} \cdot \sqrt[3]{20y}$

3. $\sqrt[3]{36x^2y^5} \cdot \sqrt[3]{-6x^2y}$

4. $5\sqrt{7x^3y} \cdot \sqrt{28y^2}$

5. $-\sqrt[3]{9x^5y^2} \cdot \sqrt[3]{2x^2y^5}$

6. $\sqrt{3}(\sqrt{12} - \sqrt{21})$

Problem

What is the simplest form of $\frac{\sqrt{9y}}{\sqrt{2x}}$?

Rationalize the denominator and simplify. Assume that all variables are positive.

$$\frac{\sqrt{9y}}{\sqrt{2x}} = \frac{\sqrt{9y}}{\sqrt{2x}}$$

Rewrite as a square root of a fraction.

$$= \frac{\sqrt{9y \cdot 2x}}{\sqrt{2x \cdot 2x}}$$

Make the denominator a perfect square.

$$= \sqrt{\frac{18xy}{4x^2}}$$

Simplify.

$$= \frac{\sqrt{18xy}}{\sqrt{2^2 \cdot x^2}}$$

Write the denominator as a product of perfect squares.

$$= \frac{\sqrt{18xy}}{2x}$$

Simplify the denominator.

$$= \frac{\sqrt{3^2 \cdot 2 \cdot x \cdot y}}{2x}$$

Simplify the numerator.

$$= \frac{3\sqrt{2xy}}{2x}$$

Use $\sqrt[n]{a^n} = a$ to simplify.

Exercises

Rationalize the denominator of each expression. Assume that all variables are positive.

7. $\frac{\sqrt{5}}{\sqrt{x}}$

8. $\frac{\sqrt[3]{6ab^2}}{\sqrt[3]{2a^4b}}$

9. $\frac{\sqrt[4]{9y}}{\sqrt[4]{x}}$

10. $\frac{\sqrt{10xy^3}}{\sqrt{12y^2}}$

11. $\frac{4\sqrt[3]{k^9}}{16\sqrt[3]{k^5}}$

12. $\sqrt{\frac{3x^5}{5y}}$

13. $\frac{\sqrt[4]{10}}{\sqrt[4]{z^2}}$

14. $\sqrt[3]{\frac{19a^2b}{abc^4}}$