

### 3-6 Solving Systems Using Matrices

- A matrix is a combination of rows and columns used to arrange data. These arrangements can be mathematically manipulated to solve problems.
- The size or dimensions of a matrix are always reported as number of rows by number of columns (rows X columns). Rows are horizontal ( $\leftrightarrow$ ), while columns are vertical ( $\updownarrow$ ). Below, matrix A is 2 X 3 and matrix B is 3 X 4.
- When finding an element in a matrix, use the name of the matrix followed by subscripts listing the row and column of the entry. Number 1 below,  $A_{13}$ , wants to know which entry is in the 1<sup>st</sup> row and 3<sup>rd</sup> column of matrix A. The answer is 8.

$$0x + 6y + 3z = 2$$

$$6y + 3z = 2$$

- Below, Matrix B represents the system:  $4x + 5y + 1z = 13$  which is the same as  $4x + 5y + z = 13$ .

$$2x + 2y + 0z = -10$$

$$2x + 2y = -10$$

Identify the indicated element.

$$A = \begin{bmatrix} 3 & 5 & 8 \\ 4 & 1 & 6 \end{bmatrix} \qquad B = \begin{bmatrix} 0 & 6 & 3 & 2 \\ 4 & 5 & 1 & 13 \\ 2 & 2 & 0 & -10 \end{bmatrix}$$

- |             |             |             |             |
|-------------|-------------|-------------|-------------|
| 1. $A_{13}$ | 2. $B_{24}$ | 3. $B_{12}$ | 4. $A_{22}$ |
| 5. $B_{31}$ | 6. $A_{21}$ | 7. $B_{23}$ | 8. $A_{11}$ |

Write a matrix to represent each system.

- |  |   |  |
|--|---|--|
| 9. $\begin{cases} 3x + y = -4 \\ -2x + 4y = 7 \end{cases}$ | 10. $\begin{cases} 6x = 11 \\ -3x + 4y = 2 \end{cases}$ | 11. $\begin{cases} 4x - y + 2z = 10 \\ 5x + 2y - 3z = 0 \\ x - 3y + z = 6 \end{cases}$ |
|--|---|--|

Write the system of equations represented by each matrix.

- |  |   |   |
|--|---|---|
| 12. $\begin{bmatrix} 2 & 5 & 0 & 13 \\ -3 & 1 & 2 & 6 \\ 4 & 0 & -3 & 5 \end{bmatrix}$ | 13. $\begin{bmatrix} 2 & 1 & -7 \\ 0 & 4 & 9 \end{bmatrix}$ | 14. $\begin{bmatrix} 6 & 4 & -2 & 17 \\ 1 & -5 & 2 & -10 \\ 0 & 3 & -1 & 0 \end{bmatrix}$ |
|--|---|---|

**Graphing Calculator Solve each system.**

15. 
$$\begin{cases} 4x - y = 10 \\ -2x + 5y = 4 \end{cases}$$

16. 
$$\begin{cases} x - 2y + 3z = 18 \\ 9x + 2y - z = -2 \\ -6x - y + 2z = 4 \end{cases}$$

17. 
$$\begin{cases} 3x - 4y + z = 15 \\ -2x - 6y + 3z = 4 \\ 2x + 2y - 2z = -1 \end{cases}$$

18. 
$$\begin{cases} 4x + y - 2z = 3 \\ 2y + z = 4 \\ 3x - 3y - z = 9 \end{cases}$$

19. 
$$\begin{cases} 5x - 2y + z = -1 \\ -x - y - 2z = 5 \\ 3x + 2y + 2z = 2 \end{cases}$$

20. 
$$\begin{cases} 3x + 5z = -4 \\ -2x + y - 3z = 9 \\ -x - 2y + 9z = 0 \end{cases}$$

21. Suppose the movie theater you work at sells popcorn in three different sizes. A small costs \$2, a medium costs \$5, and a large costs \$10. On your shift, you sold 250 total containers of popcorn and brought in \$1726. You sold twice as many large containers as small ones.

System: 
$$\left\{ \begin{array}{l} \\ \\ \\ \end{array} \right.$$

- a. How many of each popcorn size did you sell?
- b. How much money did you bring in from selling small size containers?

22. **Open Ended** Write a matrix for a system of equations that does not have a unique solution. (Hint: Recall what a system graph of two lines looked like that had infinitely many solutions or no solutions.)

System: 
$$\left\{ \begin{array}{l} \\ \\ \\ \end{array} \right.$$

23. The following matrix shows the prices passengers on an airline flight paid for a recent ticket and how many passengers were on that flight. Some passengers paid full price for their tickets, and some bought their tickets during a half-price sale. How many passengers bought each price of ticket?

$$\left[ \begin{array}{cc|c} 1 & 1 & 100 \\ 120 & 240 & 20,160 \end{array} \right]$$

24. **Error Analysis** Your friend says that the matrix below represents the system of equations. What error did your friend make? What is the correct system of equations?

$$\left[ \begin{array}{ccc|c} 4 & 0 & -1 & 4 \\ -3 & 2 & -2 & -2 \\ 1 & -3 & -2 & -6 \end{array} \right] \quad \begin{cases} 4x + y - z = 4 \\ -3x + 2y - 2z = -2 \\ x - 3y - 2z = -6 \end{cases}$$